

Topic Exam

Calculus

Video Solutions to this exam can be found at:

www.revisionvillage.com/calculus-exam

Question 1

[Maximum mark: 6]



Let $f'(x) = 3x^2 + 2x - 1$. Given that f(2) = 5, find f(x).



Question 2

[Maximum mark: 6]



Let $f(x) = e^{3x}$. The line L is the tangent to the curve of f at (0, 1).

Find the equation of *L* in the form y = mx + c.

Question 3

[Maximum mark: 6]

Consider $f(x) = x^3 - \frac{p}{x}$, $x \neq 0$, where p is a constant.

(a) Find f'(x). [3]

(b) There is a minimum value of f(x) when x = 1. Find the value of p. [3]

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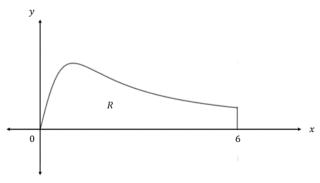


Question 4

[Maximum mark: 6]



The following diagram shows the graph of $f(x) = \frac{4x}{x^2+1}$, for $0 \le x \le 6$, and the line x = 6.



Let R be the region enclosed by the graph of f, the x-axis and the line x = 6.

Find the area of R.



Question 5

[Maximum mark: 7]



Let
$$f(x) = \frac{\ln x}{x}$$
, for $x > 0$.

- (a) Find f'(x). [3]
- (b) The graph of f has a maximum at point P. Find the coordinates of P. [4]



Question 6

[Maximum mark: 6]



Let
$$f(x) = e^{-3x}$$
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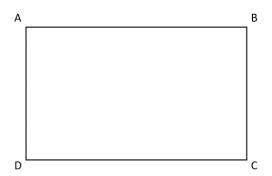
- (a) Write down f'(x), f''(x) and f'''(x).
- (b) Find an expression for $f^{(n)}(x)$. [3]

[3]

Question 7

[Maximum mark:	7]	0000 0000 0000 0000
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A farmer wants to build a rectangular enclosure for his chickens. The area of the enclosure must be 350m². The fencing used for the side AB costs \$13 per metre. The fencing for the other three sides costs \$4 per metre. The farmer wants the cost of the enclosure to be a minimum. Find the minimum cost.



Question 8

[Maximum mark: 7]



Let
$$f(x) = g(x) \times h(x)$$
, where $g(3) = 6$, $h(3) = 2$, $g'(3) = 4$ and $h'(3) = 1$.

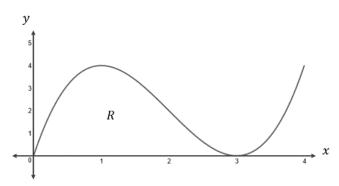
Find the equation of the normal to the graph of f at x = 3.

Question 9

[Maximum mark: 15]



Let $f(x) = x(x-3)^2$, for $0 \le x \le 4$. The following graph shows f.



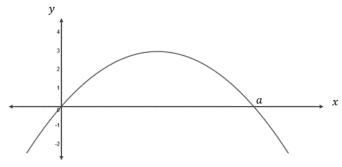
Let R be the region enclosed by the x-axis and the curve of f.

(a) Find the area of R.

(b) Find the volume of the solid formed when R is rotated 360° about the x-axis. [4]

[4]

(c) The diagram below shows part of the graph of the quadratic function g(x) = x(a-x). The graph of g crosses the x-axis when x = a.



The area enclosed by the graph of g is equal to the area of R. Find the value of a.

Question 10

[Maximum mark: 15]



Let $f(x) = \sqrt{2x+1}$ for $x \ge -0.5$.

- (a) Find
 - (i) f(12);
 - (ii) f'(12).

[4]

Consider another function g(x). Let P be a point on the graph g. The x-coordinate of P is 12. The equation of the tangent to the graph at P is y = x + 3.

- (b) Write down g'(12). [2]
- (c) Find g(12). [2]
- (d) Let $h(x) = f(x) \times g(x)$. Find the equation of the tangent of h at the point where x = 12.