Topic Exam

Algebra

Video Solutions to this exam can be found at:

www.revisionvillage.com/algebra-exam

Question 1

[Maximum mark: 7]

An arithmetic sequence is given by 3, 5, 7, …

(a)   Write down the value of d.   [1]

(b)   Find.

      (i) u_{10}.

      (ii) s_{10}.   [4]

(c)   Given that $u_n = 253$, find the value of n.   [2]

Question 2

[Maximum mark: 7]

Find the value of each of the following, giving your answer as an integer.

(a) $\log_{10}(100)$.   [2]

(b) $\log_{10}(50) + \log_{10}(2)$.   [2]

(c) $\log_{10}(4) - \log_{10}(40)$.   [3]

Question 3

[Maximum mark: 6]

Consider the expansion of $(2x - 1)^9$.

(a)   Write down the number of terms in this expansion.   [1]

(b)   Find the coefficient of the term in x^2.   [5]

Question 4

[Maximum mark: 6]

In an arithmetic sequence, $u_5 = 24$, $u_{13} = 80$.

(a)   Find the common difference.   [2]

(b)   Find the first term.   [2]

(c)   Find the sum of the first 20 terms of the sequence.   [2]

Question 5

[Maximum mark: 6]

(a)   Write down the value of.

      (i) $\log_2 8$

      (ii) $\log_5 \left(\frac{1}{25}\right)$

      (iii) $\log_9 3$ [3]

(b)   Hence, solve $\log_2 8 + \log_5 \left(\frac{1}{25}\right) + \log_9 3 = \log_{16} x$.   [3]

Question 6

[Maximum mark: 6]

Find the value of

(a) $\log_7 98 - \log_7 2$;   [2]

(b) $49^{\log_7 6}$.   [4]

Question 7

[Maximum mark: 7]

In the expansion of $(2x + 1)^n$, the coefficient of the term in x^2 is $40n$, where $n \in \mathbb{Z}^+$. Find n.

Question 8

[Maximum mark: 6]

Find the values of x when $25^{x^2-2x} = \left(\frac{1}{125}\right)^{4x+2}$.

Question 9

[Maximum mark: 14]

The first two terms of an infinite geometric sequence, in order, are

$$3\log_3 x, \ 2\log_3 x, \text{ where } x > 0.$$

(a)   Find r.   [2]

(b)   Show that the sum of the infinite sequence is $9log_3 x$.   [3]

The first three terms of an arithmetic sequence, in order, are

$$\log_3(x), \ \log_3\left(\frac{x}{3}\right), \ \log_3\left(\frac{x}{9}\right), \text{ where } x > 0.$$

(c)   Find d, giving your answers as an integer.   [3]

Let S_6 be the sum of the first 6 terms of an arithmetic sequence.

(d)   Show that $S_6 = 6\log_3(x) - 15$.   [3]

(e)   Given that S_6 is equal to one third of the sum of the infinite geometric   [3]
        sequence, find x, giving your answer in the form of 3^P.

Question 10

[Maximum mark: 15]

The first three terms of an infinite geometric sequence are k-4, 4, k+2, where k∈ ℤ.

(a)   (i)   Write down an expression for *r*.

        (ii)   Hence, show that k satisfies the equation $k^2 - 2k - 24 = 0$ [5]

(b)   (i)   Find the possible values for k.

        (ii)   Find the possible values for *r*.   [5]

(c)   The geometric sequence has a finite sum.

        (i)   Which value of r leads to this sum. Justify answer.

        (ii)   Find the sum of the sequence.   [5]