Unit 5, Test 1 Review– Exponential and Logarithmic Functions

1) Jack evaluated and solved the following logarithm problems. Some are correct and some are incorrect. Analyze each problem Jack did. If the problem is correct, justify the process Jack used. If the answer is incorrect, then explain what he did wrong and correct the mistake if possible. (3 points each)

a) Evaluate: $\log_2(\frac{1}{16})$ $\quad$ $2^? = 16$	b) Solve: $3 = \log_x 64$ $\quad$ $x^3 = 64$	c) Solve: $m = \log_4(32)$ $\quad$ $m^4 = 32$
a) Answer: 4	b) Answer: $x = 4$	c) Answer: $m \approx 2.378$
Wrong $$2^? = \frac{1}{16}$$ $$? = -4$$	Correct	Wrong $$4^m = 32$$ $$2^{2m} = 2^5$$ $$m = 5/2$$

2) Write the expression as an exponential expression:
   a. $\log_9 5 = 2x$

   $$9^{2x} = 5$$

   b. $\log(x+4) = 5x$

   $$10^{5x} = x + 4$$

3) Write the expression as a logarithmic expression:
   a. $y^{3-x} = 7$

   $$\log_y 7 = 3 - x$$

   b. $7^x = 12$

   $$\log_7 12 = x$$

4) Condense the expressions:
   a. $\frac{1}{2}\log_5 x + \log_5 y + \frac{1}{3}\log_5 64$

   $$\log_5 (x^{1/2} y \cdot 4)$$

   b. $4\log 2 - \log 3 + \log 4$

   $$\log\left(\frac{2^4 \cdot 4}{3}\right)$$

   $$\log\left(\frac{64}{3}\right)$$

5) Expand the expressions:

   a. $\log(4a^3b^4)$

$$\log 4 + 3\log a + 4\log b$$

   b. $\log_x \frac{\sqrt{wx}}{y^3}$

$$\tfrac{1}{2}\log_x w + \log_x x - 3\log_x y$$
$$\tfrac{1}{2}\log_x w + 1 - 3\log_x y$$

6) Evaluate the logarithm:

   a. $\log_3 81 = \underline{\quad 4 \quad}$

   b. $\log_{1/3} 27 = \underline{\quad -3 \quad}$

   c. $\log_2 \left(\frac{1}{32}\right) = \underline{\quad -5 \quad}$

7) Solve the exponential equation:

   a. $5^{4x-5} = 5^{2x+15}$

$$4x - 5 = 2x + 15$$
$$2x = 20$$
$$x = 10$$

   b. $2^{x+1} = \left(\frac{1}{64}\right)^{x+1}$

$$2^{x+1} = 2^{-6x-6}$$
$$x + 6 = -6x - 6$$
$$7x = -12$$
$$x = -12/7$$

8) Solve the logarithmic equation. Remember to check for extraneous solutions.

   a. $\log(2x - 1) = \log(-4x - 13)$

$$2x - 1 = -4x - 13$$
$$6x = -12$$
$$x = -2$$

   b. $\log(x - 21) - 1 = 2$

$$\log(x - 21) = 3$$
$$10^3 = x - 21$$
$$1000 = x - 21$$
$$x = 1021$$

   c. $\log_3(x + 1) - \log_3 x = 1$

$$\log_3(x^2 + x) = 1$$
$$3^1 = x^2 + x$$
$$0 = x^2 + x - 3$$
$$x = \frac{-1 \pm \sqrt{1 - 4(-3)}}{2}$$
$$x = \frac{-1 \pm \sqrt{13}}{2}$$
$$x = \frac{-1 + \sqrt{13}}{2}$$

   d. $\log_6 x + \log_6(x + 5) = 2$

$$\log_6(x^2 + 5x) = 2$$
$$6^2 = x^2 + 5x$$
$$0 = x^2 + 5x - 36$$
$$0 = (x + 9)(x - 4)$$
$$x = -9, 4$$

9) Simplify each expression, show supporting work below:

   a. $\left(\frac{2x^3y^6}{y^5}\right)^3$

   $8x^9y^3$

   b. $\frac{2x^6y^{-1}}{3xy^3} \cdot \frac{5x^{-7}y^3}{15x^2y^{-4}}$

   $\frac{2y^3}{9x^4}$

10) Solve the equations (round to three decimal places if necessary):

   a) $10^x + 3 = 4$

   $10^x = 1$

   $x = 0$

   b) $2^{x+5} + 3 = 12$

   $2^{x+5} = 9$

   $\log_2 9 = x + 5$

   $x = -1.830$

   c) $4^{5x-2} = 14$

   $\log_4 14 = 5x - 2$

   $x = 0.781$

   d) $9\log_4 x = 54$

   $\log_4 x = 6$

   $4^6 = x$

   $x = 4096$

11) Approximate the value of $\log_{12}(25)$ to the nearest hundredth.

   1.30